Let f the function defined by $f(x) = x + \ln 4 + \frac{2}{1+e^x}$ and (C) its representative curve in an orthonormal system $\left(O;\vec{i};\vec{j}\right)$.

1) Justify that the domain of f is $\mathbb{R}$.
2) Calculate $\lim_{x\to+\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$.
3) Calculate for all real numbers x , $f(x) + f(-x)$. Deduce that A (0; 1+ln4) is a center of symmetry.
4) Show that f(x) = $x + 2 + \ln 4 - \frac{2e^x}{1+e^x}$ then deduce that the straight lines (d) and (d') of respective equations y = x + ln4 and y = x + 2 +ln4 are asymptotes to (C).
5) Show that (C) lies between (d) and (d').
6) Show that (C) admits an inflection point whose coordinates are to be determined.
7) Draw the table of variations of f then trace (C) , (d) and (d').
8) Show that the equation f(x) = 3 admits a unique root α and that $1.1 \prec \alpha \prec 1.2$
9) Show that f admits over $\mathbb{R}$ an inverse function f^{-1}.
10) Without finding the expression of the inverse:
    a) Solve $f^{-1}(x) = f(x)$
    b) Trace the representative curve of the function f^{-1}.
11) Consider a real positive number β.
    a) What does the integral $I(\beta) = \int_0^{\beta} \left[f(x) - x - \ln 4\right] dx$ represent?
    b) Show that $I(\beta) = 2\ln\left(\frac{2e^{\beta}}{e^{\beta}+1}\right)$.

III- (18pts)

Consider the function f defined over $]0;+\infty[$ by $f(x)=\ln\left(\frac{x+1}{x}\right)-\frac{1}{x+1}$ and let (C) be its representative curve in an orthonormal system $(o,\vec{i},\vec{j})$ (graphical unit is 5cm).

1) a) Determine $\lim\limits_{x\to+0} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$

b) Deduce the existence of 2 asymptotes.

2) a) Calculate $f'(x)$ and show that $f'(x)=-\frac{1}{x(x+1)^2}$. Deduce the sign of $f'(x)$.

b) Set up the table of variations of f and deduce the sign of $f(x)$ $\forall\ x\in\ ]0;+\infty[$.

c) Draw (C).

3) Consider the function g defined over $[0;+\infty[$ by $g(x)=\begin{cases} x\ln\left(\frac{x+1}{x}\right) & if\ \ x>0 \\ 0 & if\ \ x=0 \end{cases}$

a) Prove that g is continuous at x = 0.

b) Study the differentiability of g at x = 0.

4) a) Calculate $\lim\limits_{x\to+\infty} g(x)$. Deduce the existence of a horizontal asymptote

b) Calculate $\forall$ x > 0, g'(x) and show that g'(x) = $f(x)$

c) Set up the table of variations of g over $[0;+\infty[$.

d) Draw the representative curve (Γ) of g in an orthonormal system $(o,\vec{i},\vec{j})$ (graphical unit 5cm).

5) a) Prove that g admits over $[0;+\infty[$ an inverse function g^{-1}. Determine the domain of definition of g^{-1} and set up the table of variations of g^{-1}.

b) Draw in the same system of (Γ), the representative curve (Γ') of g^{-1}.

c) Prove that the 2 curves (Γ) and (Γ') intersect in 2 points whose coordinates are to be determined.

III- (9points)

Consider the function f defined over IR by: $f(x) = \dfrac{4e^x}{e^x + 1}$. ($C_f$) represents the curve of f in an orthonormal system. (Graphical unit 2cm).

Part A:

1) a) Determine the limits of f at $-\infty$ and at $+\infty$.
   b) Deduce the equations of the asymptotes to (C_f).
2) study the variations of f, and set up the table of variations.
3) a) Prove that the point A (0; 2) is a center of symmetry of (C_f).
   b) Write the equation of the tangent (T) to (C_f) at A.
   c) Trace the asymptotes of (C_f), (T) and (C_f).

Part B:

Let F be the antiderivative of f such that F (0) = 0, let (Γ) denote its curve in a new orthonormal system. (Graphical unit 2 cm).

1) Without calculating F(x), what is the sense of variations of F ? Justify.
2) a) Justify that $F(x) = 4\ln\left(\dfrac{e^x + 1}{2}\right)$
   b) Determine $\lim\limits_{x \to +\infty} F(x)$ and $\lim\limits_{x \to -\infty} F(x)$.
   c) Deduce that (Γ) admits a horizontal asymptote whose equation is to be determined.
   d) Show that the straight line of equation: $y = 4x - 4\ln 2$ is an asymptote to (Γ).
3) Set up the table of F. Trace (Γ).

Part C:

1) Calculate, in cm^2, the area of the domain limited by (C_f), x'ox and the two straight lines of equations: $x = 0$ and $x = \ln 3$.
2) Deduce, in cm^2, the area of the portion of the plane bounded by (C_f) and the straight lines of equations: $y = 4$, $x = 0$ and $x = \ln 3$.

III) Consider the function f defined over $]-\infty, 0[\cup]0, +\infty[$ by $f(x) = x - 1 - \frac{4}{e^x - 1}$.

Designate by (C) the representative curve of f in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Show that the axis of ordinates is an asymptote to (C).

b- Calculate $\lim_{x \to +\infty} f(x)$ and prove that the line (d) of equation $y = x - 1$ is an asymptote to the curve (C).

c- Prove that the line (D) of equation $y = x + 3$ is an asymptote to (C) at $-\infty$.

2) Prove that the point S(0 ; 1) is a center of symmetry of (C).

3) a- Calculate $f'(x)$ and set up the table of variations of f.

b- Show that the equation $f(x) = 0$ has two roots α and β and verify that : $1.7 < \alpha < 1.8$ and $-3.2 < \beta < -3.1$.

4) Draw (d), (D) and (C).

5) a- Prove that $f(x) = x + 3 - \frac{4e^x}{e^x - 1}$.

b- Calculate the area of the region bounded by the curve (C), the axis of abscissas and the two lines of equations $x = 2$ and $x = 3$.

6) Let g be the inverse function of f on $]0, +\infty[$.
Prove that the equation $f(x) = g(x)$ has no roots.

IV) Let f be the function defined on IR by $f(x) = (x - 1)e^x + 1$ and designate by (C) its representat curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

1) a- Calculate $\lim_{x \to -\infty} f(x)$ and deduce an asymptote (d) of (C).

b- Study, according to the values of x, the relative positions of (C) and (d).

c- Calculate $\lim_{x \to +\infty} f(x)$ and find f(2) in decimal form.

2) Calculate $f'(x)$ and set up the table of variations of f.

3) Prove that the curve (C) has a point of inflection W whose coordinates are to be determined.

4) a- Draw (d) and (C).

b- Discuss graphically, according to the values of the real parameter m, the number of solutions of the equation $(m - 1)e^{-x} = x - 1$.

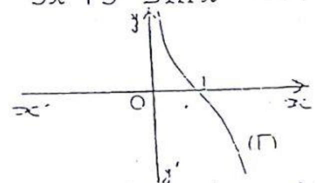
5) Calculate the area of the region bounded by (C), the axis of abscissas and the two lines of equations $x = 0$ and $x = 1$.

6) a- Show that the function f has on $[0; +\infty[$ an inverse function g and draw (G), the representati curve of g in the system $(O; \vec{i}, \vec{j})$.

b- Find the area of the region bounded by (G), the axis of ordinates and the line (d).

Exersise IV : (8 points)

Let g be a function defined over] 0 , +∞ [by : $g(x) = -3x^2+3-2\ln x$ and (Γ) be its representative curve whose graph is given to the right :



1) Calculate g(1) . Using the graph (Γ) give the sign of g(x) over] 0 , +∞ [.

2) Let f be the function defined over] 0 , +∞ [by :

$$f(x) = \frac{2\ln x-1-3x^2}{2x}$$, and (C) be its representative curve in an orthonormal system $(O , \vec{i} , \vec{j})$

a) Calculate the limits of f at the endpoints of] 0 , +∞ [.

b) Show that the straight line (Δ) of equation 2y+3x=0 is an oblique asymptote to (C) at +∞ . Determine the coordinates of the point of intersection A of (C) and (Δ) , then study the relative position of (C) and (Δ) when x ε] 0 , +∞ [.

c) Prove that $f'(x) = \frac{g(x)}{2x^2}$, then deduce the sign of $f'(x)$ over] 0 , +∞ [.

d) Set up the table of variations of f .

e) Draw (C) and (Δ).

3) Let λ be a real number which is an element of [$\sqrt{e}$, +∞ [.

a) Calculate in unit area and in terms of λ , the area $A(\lambda)$ of the domain of the plane limited by $x=\sqrt{e}$, $x=\lambda$, (C) , and (Δ).

b) What can you say about $\lim_{\lambda \to +\infty} A(\lambda)$?

Logarithm

I- Consider the function f defined by $f(x)=\frac{1+m\ln x}{x}$ where m is a real parameter.

Designate by (C_m) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) Show that the curves (C_m) pass through a fixed point A.
2) Show that the curves (C_m) admit the same asymptotes for every value of m.
3) a- Determine m so that (C_m) cuts the x-axis at the point of abscissa $x_o=\frac{1}{e}$.

   b- Study the variations of the function thus obtained and draw its curve (C).
4) Calculate the area limited by (C), the x-axis and the lines of equations $x=\frac{1}{e}$ and $x=1$.

II- Let f be a function defined over $]0;+\infty[$ by: $f(x)=2x-1+\ln\frac{x}{x+1}$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i};\vec{j})$.

1) Show that the curve (C) admits the straight line (D) of equation $y=2x-1$ as an oblique asymptote.
2) Study the position of (C) with respect to (D).
3) Show that the equation $f(x)=0$ admits a unique solution α so that $\alpha\in[0.8;0.9]$
4) Study the variations of f and draw (C).
5) Show that f admits over $]0;+\infty[$ an inverse function f^{-1}, determine its domain and construct its graph (C').
6) Construct the graph of f' the derivative of f.

III- In an orthonormal system, consider the function ~~f~~ defined over $]1;+\infty[$ by:

$f(x)=ax+\frac{b}{\ln x}$, and let (C) be its curve.

1- Determine a and b such that (C) cuts the x- axis at the point E of abscissa e and that the tangent to (C) at the point E is parallel to the straight line of equation $y=2x$.

2- Let $a=1$ and $b=-e$ then $f(x)=x-\frac{e}{\ln x}$, and let (C) be its curve.

a) Calculate the limits of $f(x)$ at 1 and $+\infty$.

b) Study the variations of $f(x)$ over $]1;+\infty[$.

c) Show that the straight line (D) of equation $y=x$ is an asymptote to (C).

d) Study the relative position of (D) and (C).

e) Give the equation of the tangent (T) to (C) at the point of abscissa e.

f) Construct (D), (T) and (C).