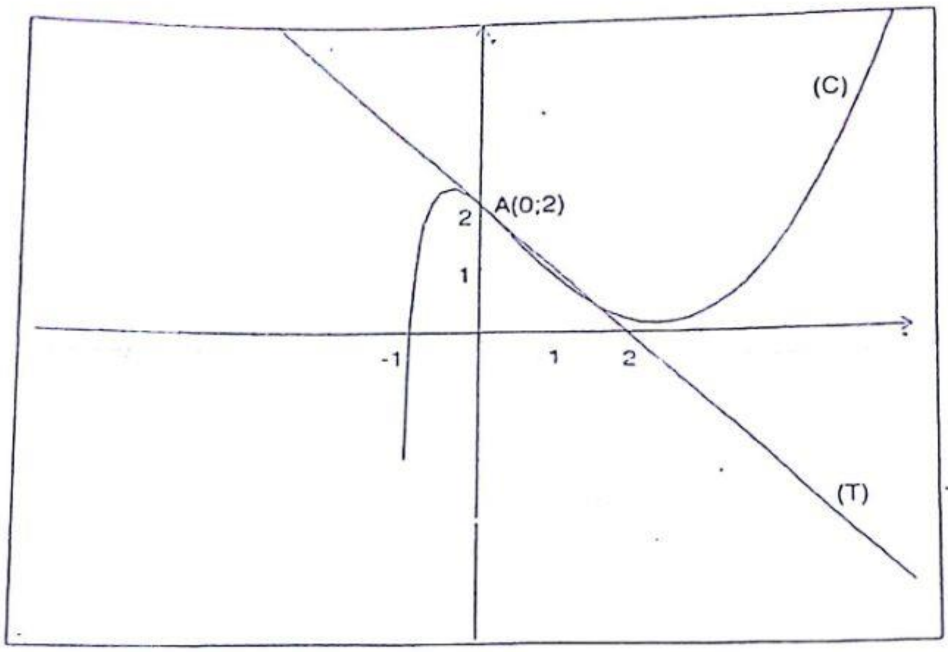
..milich College	Mathematics	Date:---/02/2006
Class: Life Science	Midyear Exam	Time:120 min

إرشادات عامة:- يجب أن يكون مع التلميذ أدوات الرسم الهندسي وآلة حاسبة غير مبرمجة.

- يستطيع التلميذ أن يجيب على المسائل بالترتيب الذي يريده.
- إن للوضوح والترتيب أهمية خاصة لذلك ننصح التلميذ بالكتابة بشكل واضح ومرتب مع تجنب التشطيب.
- عدد الأسئلة ثلاثة وجميعها إلزامية.
- العلامة القصوى 20 .

I)



Part A:

The above curve (C) is the representative curve of a function F defined over $]-1,+\infty[$ by: $F(x)= ax^2 + bx + c + 2\ln(x+1)$ and (T) is the tangent to (C) at A(0,2).

1) Calculate in terms of a , b , and c ,the derivate F ' of F . (0.5pt)
2) Use the above curve to find F(0) and F '(0). (0.5pt)
3) If $F'(3)=\frac{1}{2}$,calculate a , b , and c . (1 pt)

Part B:

Consider the function f defined over [0,5] by : $f(x) = \frac{1}{2}x^2 - 3x + 2 + 2\ln(x+1)$.

1) a) Calculate the derivate f ' of f . (0.5pt)

   b) Study the sign of f '(x) over [0,5]. Deduce the sense of variations of f. (0.75pt)

2) Consider the function g defined by: $g(x) = (x+1)\ln(x+1) - x$.

   a) Calculate the derivative g ' of g then deduce a primitive of f over [0,5]. (1 pt)

   b) Calculate $\int_0^5 f(x)dx$. (0.75pt)

II) Part A:

Consider the function g defined over] 0, +∞ [by : $g(x) = -2\ln x - xe + 1$.

1) Determine the limits of g at 0 and at +∞ . (0.75pt)
2) Study the sense of variations of g over] 0, +∞ [. (0.75pt)
3) Show that the equation g(x) = 0 admits a unique solution α in the interval [0.5 , 1] , then show that $0.6<\alpha<0.7$. (0.75pt)
4) Deduce the sign of g(x) on] 0, +∞ [. (0.5pt)

Part B:

Given the function f defined over] 0, +∞ [by: $f(x) = \dfrac{\ln x + xe}{x^2}$ and designate by (c) its representative curve in an orthonormal system $(o, \vec{i}, \vec{j})$ (graphical unit is 2 cm).

1) a) Determine the limits of f at 0 and at +∞. (0.75pt)
   b) Deduce that (c) admits two asymptotes to be determined. (0.75pt)
2) a) Calculate the derivate f ' of f and prove that $f'(x) = \dfrac{g(x)}{x^3}$. (0.75pt)
   b) Deduce the table of variations of f . (0.5pt)
3) Show that $f(\alpha) = \dfrac{1+\alpha e}{2\alpha^2}$. construct the curve (c) representative of f . (1 pt)

Part C:

Let $I_n = \displaystyle\int_{e^n}^{e^{n+1}} \frac{\ln t}{t^2}dt$ and $A_n = \displaystyle\int_{e^n}^{e^{n+1}} f(t)dt$ where n is a natural integer.

1) Using integration by parts ,show that : $I_n = \dfrac{n+1}{e^n} - \dfrac{n+2}{e^{n+1}}$. (0.75pt)
2) a) Show that $A_n = I_n + e$. (0.75pt)
   b) Calculate I_0 and A_0 . (0.5pt)
   c) Give a geometric interpretation of A_0. (0.5pt)

European Lebanese School

Problems on logarithm functions

I - f is a function defined on $]0, +\infty[$ by $f(x) = x\ln(x^2) - 2x$.

1) Show that $f(x) = 2x.\ln\left(\frac{x}{e}\right)$.
2) a) Find the limit of f(x) as $x \to +\infty$ and as $x \to 0^+$.
   b) Calculate $f'(x)$.
   c) Set up the table of variations of f.
   d) Solve the equation $f(x) = 0$.
3) Show that the equation $f(x) = 2$ admits a unique solution on $[1, 5]$.
4) Draw the graph of f in an orthonormal system.

II - Consider the function f defined on $]0 ; +\infty[$ by $f(x) = \frac{5\ln x}{\sqrt{x}}$. Let (C) its curve in an orthonormal system $\left(O ; \vec{i} ; \vec{j}\right)$ (1 unit = 1cm).

1) Study the limits of f at $+\infty$ and at 0^+. What can you deduce about (C) ?
2) Set up the table of variation of f.
3) Find the equation of the tangent (T) to (C) at the point A of abscissa 1.
4) Draw (T) and (C).

III - Consider the function g defined by : $g(x) = -x^2 + 1 - \ln x$.

1) Calculate $\lim\limits_{x \to 0^+} g(x)$ and $\lim\limits_{x \to +\infty} g(x)$. Set up the table of variation of g.
2) Calculate g(1). Deduce the sign of g(x).
3) Consider the function $f(x) = -\frac{1}{2}x + 1 + \frac{\ln x}{2x}$. Let (C) be its representative curve in an orthonormal system $(O ; \vec{i} ; \vec{j})$.

a - Express $f'(x)$ in terms of g(x). Calculate $\lim\limits_{x \to +\infty} f(x)$, $\lim\limits_{x \to 0^+} f(x)$ and $\lim\limits_{x \to +\infty}(f(x) + \frac{1}{2}x - 1)$.
   Deduce the asymptotes to (C).

b - Set up the table of variation of f.

c - Discuss the sign of $f(x) + \frac{1}{2}x - 1$. Deduce the position of (C) with respect to the straight line (d) : $y = \frac{-x}{2} + 1$.

4) Draw (d) and (C).

IV - Consider the function f defined on $]0 ; +\infty[$ by $f(x) = x^2 + 6\frac{\ln x}{x}$. Denote by (φ) its representative curve in an orthogonal system. (on the axis of abscissas take 1 unit = 2cm, on the axis of ordinates take 1 unit = 0.5)

1) Calculate the limits of f at 0^+ and at $+\infty$.
2) By studying the variation of the function g defined on IR^{+*} by : $g(x) = x^3 + 3 - 3\ln x$, show that, for every $x > 0$, $g(x) > 0$.
3) Study the variation of f.
4) Write the equation of the tangent (T) to (φ) at the point of (T) of abscissa 1.
5) Draw (φ).

EXERCICES ON LOGARITHMIC FUNCTIONS (041108)

1 Consider the function f defined by : $f(x) = x + \ln\left(\frac{x}{2x+1}\right)$ and designate by (C) its curve in an orthonormal system $(O ; \vec{i} ; \vec{j})$ (1unit = 1cm).

1) Show that the domain of definition of f is $]-\infty, \frac{-1}{2}[\cup]0, +\infty[$.
2) Determine the limits of f at $-\infty$, $\frac{-1}{2}$, 0 and $+\infty$.
3) Deduce that (C) admits 2 asymptotes whose equations are to be determined.
4) Set up the table of variation of f.
5) Show that the line (Δ) whose equation is $y = x - \ln 2$ is third asymptote to (C). Study the relative position of (C) and (Δ).
6) Show that the equation $f(x) = 0$ admits a unique solution $\alpha \in]0; +\infty[$ and that $\alpha \in \left]1, \frac{5}{4}\right[$.
7) Draw (Δ) and (C).

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2 Consider the function f defined on $]0 ; +\infty[$ by $f(x) = \frac{5 \ln x}{\sqrt{x}}$. Let (C) its curve in an orthonormal system $\left(O ; \vec{i} ; \vec{j}\right)$ (1 unit = 1cm).

a) Study the limits of f at $+\infty$ and at 0^+. What can you deduce about (C) ?
b) Set up the table of variation of f.
c) Find the equation of the tangent (T) to (C) at the point A of abscissa 1.
d) Draw (T) and (C).

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3 Consider the function f defined on $]0 ; +\infty[$ by $f(x) = x^2 + 6\frac{\ln x}{x}$. Denote by ($\varphi$) its representative curve in an <u>orthogonal</u> system. (on the axis of abscissas take 1 unit = 2cm , on the axis of ordinates take 1 unit = 0.5)

a) Calculate the limits of f at 0^+ and at $+\infty$.
b) By studying the variation of the function g defined on IR^{+*} by : $g(x) = x^3 + 3 - 3\ln x$, show that, for every $x > 0$, $g(x) > 0$.
c) Study the variation of f.
d) Write the equation of the tangent (T) to (φ) at the point of (T) of abscissa 1.
e) Draw (φ).

EXERCISE – I

Part A .

Let g be the function defined over IR by $g(x) = a + (b - x)e^x$ where a and b are real numbers.

Designate by (G) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

Determine a and b such that (G) has at the point $A(0;3)$ a tangent parallel to the straight line (D) of equation $y = x$.

Part B.

Let f be the function defined over IR by $f(x) = (2 - x)e^x + 1$.

Designate by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Calculate $\lim\limits_{x \to -\infty} f(x)$ and deduce an asymptote (d) to (C)

   b- Study the relative position of (C) and (d) .

   c- Calculate $\lim\limits_{x \to +\infty} f(x)$.

2) Calculate $f'(x)$ and set up the table of variations of f .

3) Show that (C) has an inflection point W .

4) Show that the equation $f(x) = 0$ has a unique root α such that

   $2.1 < \alpha < 2.2$

4) a- Draw (d) and (C)

   b- Discus graphically, according to the values of the real number m, the number of solutions of the equation $(m - 1)e^{-x} = 2 - x$.

5) Designate by $A(\alpha)$ the area of the domain bounded by (C), the x-axis and the y-axis.

   Show that $A(\alpha) = \dfrac{(3 - \alpha)^2}{\alpha - 2}$.