Beirut International School (BIS)
Life Sciences-General Sciences
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Mathematics Department
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Logarithmic Functions

Question 1

Determine the derivative of each of the following functions.

1) $f(x) = x + 1 + 2\ln x$
2) $f(x) = x^2 + 1 - \dfrac{\ln x}{2x}$
3) $f(x) = x\ln x + x + \dfrac{1}{2}$
4) $f(x) = (\ln x)^2 + \ln x - 2$
5) $f(x) = \dfrac{-1}{x+1}\ln x$
6) $f(x) = \ln\left(\dfrac{x+1}{x-1}\right)$
7) $f(x) = \dfrac{x-1}{x}\ln x$
8) $f(x) = (x-e)(\ln x - 1)$
9) $f(x) = \dfrac{x-1}{x} - 3\ln x$

Question 2

Determine each of the following limits.

1) $\lim\limits_{x \to 0^+} (x - \ln x + 2)$
2) $\lim\limits_{x \to -\infty} \ln\left(\dfrac{x+1}{x-1}\right)$
3) $\lim\limits_{x \to +\infty} (x - \ln x + 2)$
4) $\lim\limits_{x \to +\infty} \left(x^2 + 1 - \dfrac{\ln x}{2x}\right)$
5) $\lim\limits_{x \to 0^+} x(\ln x + x - 2)$
6) $\lim\limits_{x \to +\infty} \left((\ln x)^2 - \ln x - 2\right)$
7) $\lim\limits_{x \to 0^+} \left(\dfrac{x-1}{x}\ln x\right)$
8) $\lim\limits_{x \to 1^+} \dfrac{\ln x}{x-1}$
9) $\lim\limits_{x \to +\infty} \dfrac{\ln x}{x-1}$

Question 3

Consider the function f defined, on $]0,+\infty[$, by: $f(x)=\frac{x-1}{x}\ln x$. Let (C) be the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

1) Prove that $x=0$ is a vertical asymptote to (C).
2) Determine the limit of f at $+\infty$.
3) Show that $f'(x)=\frac{x-1+\ln x}{x^2}$.
4) The adjacent table is the table of variations of the function g defined, on $]0,+\infty[$, by: $g(x)=x-1+\ln x$.

x	0		1		$+\infty$
$g'(x)$	‖	$+$		$+$	
$g(x)$	‖ $-\infty$	↗	0	↗	$+\infty$

a- Study the sign of g(x) on $]0,+\infty[$.

b- Using the above table, construct the table of variations of f.

5) Determine the equation of the tangent (T) to (C) at $A\left(e,1-\frac{1}{e}\right)$.
6) Draw (T), then (C).
7)

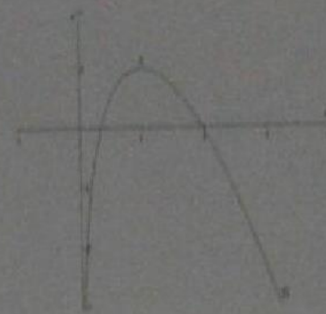
a- Prove that the function f admits, on $]0,1[$, an inverse function f^{-1}.

b- Construct the table of variations of f^{-1}.

c- Draw (G), the representative curve of f^{-1} in the same reference of (C).

Question 7

Part A



Let f be the function defined, on $]0\,,+\infty[$, by: $f(x) = ax + (bx + c)\ln x$, where a, b, and c are three real numbers. (C), the adjacent curve, is the representative curve of f. Find a, b, and c, knowing that $f(2) = 2 - 3\ln 2$, $A(1\,,1)$ is a point of (C), and A is an extremum of f.

Part B

Let g be the function defined, on $]0\,,+\infty[$, by: $g(x) = x + (1 - 2x)\ln x$. Without using the given curve:

1) Calculate the limit of g at 0 and at $+\infty$.
2)
   a- Determine the derivative function of g.
   b- Study the sign of $-2\ln x$ and that of $\frac{1-x}{x}$. Deduce the sign of $g'(x)$ and the variations of g.
3) Let (Δ) the straight-line of equation $y = x$.
   a- Solve, in $\mathbb{R}$, the equation $g(x) - x = 0$, then give a graphical interpretation of the solutions.
   b- Study the position of the representative curve of g with respect to the straight-line (Δ).

Question 8

f is the function defined, on $]0\,,+\infty[$, by: $f(x) = \frac{1}{2}x - 1 + \frac{\ln x}{x}$. Let (C) be its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1)
   a- Study the limits of f at 0 and at $+\infty$.
   b- Prove that the line (Δ): $y = \frac{1}{2}x - 1$ is an oblique asymptote to (C).
   c- Specify the relative positions of (C) and (Δ).
2)
   a- Calculate $f'(x)$ and $f''(x)$. Deduce the sense of variations of f' and the sign of $f'(x)$.
   b- Prove that (C) has an inflection point whose coordinates are to be determined.
   c- Construct the table of variations of f.
3)
   a- Show that the equation $f(x) = 0$ has a unique root α. Verify that $1.4 < \alpha < 1.5$.
   b- Verify that $\ln\alpha = \alpha - \frac{1}{2}\alpha^2$.
4) Sketch (Δ), then (C). (unit = 2cm).