Exercise 1

The plane is attached to an orthonormal system $(O;\vec{i};\vec{j})$. Let (H) be the set of points M(x,y) such that : $9x^2 - 16y^2 - 144 = 0$.

1) Show that (H) is a hyperbola and find its verticies S and S', the focii, directices and the asymptotes.
2) Trace (H).
3) Let $\theta \in \left[0, \frac{\pi}{2}\right[$ and M the point of coordinates $\left(\frac{4}{\cos\theta}; 3\tan\theta\right)$.
   a) Show that $M \in (H)$
   b) Let (T) be the tangent to (H) at M. show that $(T): 3x - 4y\sin\theta = 12\cos\theta$
   c) Let (T_S) and $(T_{S'})$ be (H) at S and S' respectively.(T) cuts (T_S) and $(T_{S'})$ at P and P' respectively. Show that $\overrightarrow{SP}.\overrightarrow{SP'} = -9$.

Exercise 2

In the plane attached to an orthonormal system $(O;\vec{i};\vec{j})$, consider the hyperbola (H): $x^2 - \frac{y^2}{4} = 1$

Designate by M the point of coordinates $\left(\frac{1}{\cos\theta}; 2\tan\theta\right)$ where $\theta \in \left]0, \frac{\pi}{2}\right[$

1) a) Determine the verices and the focii of (H).
   b) write cartesian equations of the asymptotes (Δ_1) and (Δ_2).
   c) Trace (H)
   d) verify that the point M belongs to (H)
2) Let (T_M) be the tangent to (H) ant M. show that the equation of (T_M) is: $2x - y\sin\theta - 2\cos\theta = 0$.
3) Designate by P_1 and P_2 the points of intersection of (T_M) with (Δ_1) and (Δ_2) respectively
   a) Find the coordinates of P_1 and P_2
   b) Show that the area of triangle OP_1P_2 is independent of θ.

Exercise 3

In the plane attached to an orthonormal system $(O;\vec{i};\vec{j})$, consider the parabola (P): $y^2 = 2x$ and the points $M\left(\frac{t^2}{2}, t\right)$ and $M'\left(\frac{1}{2t^2}, -\frac{1}{t}\right)$ where t is a non zero real parameter.

1) a) Determine the focus and the directrix of (P)
   b) Trace (P) and place F
   c) verify that M and M' belong to (P)
2) Designate by (T) and (T') the tangents to (P) at M and M'
   a) Show that M, M' and F are collinear.ver
   b) Write the equations of the tangents (T) and (T')
   c) Show that (T) and (T') are perpendicular
   d) Let H be the point of intersection of (T) and (T'). Show that H varies on a fixed straight line as t varies in R*

Exercise 4

In the plane attached to an orthonormal system $(O;\vec{i};\vec{j})$,consider the ellipse (E): $\frac{x^2}{4} + y^2 = 1$ and $M(2\cos\theta, \sin\theta)$, where $\theta \in \left]0, \frac{\pi}{2}\right[$.

1) a) Determine the verticies and the focii of (E)

b) Trace (E) and place the focii

c) verify that M belongs to (E)

2) Let (T_M) be the tangent to (E) at M. show that $(T_M): x\cos\theta + 2y\sin\theta - 2 = 0$.

3) Consider A(2,0) and A'(-2,0) the focal vertices of (E). Designate by (T) and (T') be the tangents to (E) at A and A' respectively and by P and P' the points of intersection of (T_M) with (T) and (T') respectively.

   a) Find the coordinate of P and P'

   b) Show that $\overrightarrow{AP}.\overrightarrow{A'P'} = 1$

Exercise 5

In the complex plane (P) given the application: f that associate to every point M(z) a point M'(z') where z is a non real complex number such that $z' = \dfrac{z^2}{z - \overline{z}}$

1) a) show that f admits no invariant point.

   b) show that for any M(z) $\in (P) - (x'Ox)$, (MM') is a fixed straight line.

2) a) show that if $z \notin \mathbb{R}$, $|z' - z| = |z'|$

   b) deduce that M' is the point of intersection of the perpendicular bisector of [OM] and the straight line (Δ) passing through M and parallel to (y'y).

3) let (D): y = 2. Show that if M moves on (D), then M' moves on a fixed parabola whose focus and directrix are to be determined.

Exercise 6

Let z be a complex number and $Z = iz^2 + 2z$ with z = x + iy, x and y are real numbers.

1) Find the algebraic form of Z in terms of x and y.

2) In the complex plane attached to an orthonormal system. Given (Γ) the set of points M of affix z such that Z is a positive real.

   a) Find the cartesian equation of (Γ).

   b) Show that (Γ) is a part of a hyperbola (H) to be determined.

Exercise 7