

الفصل الثاني التاريخ : 2015/ 3 /	امتحانات الشهادة الثانوية فرع علوم الحياة	ثانوية منارة جبل عامل المروانية
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Exercise 1 (2 points)

In the following table, only one answer to each question is correct. Write the number of each question with its answer and write a **justification**.

N°	Questions	Réponses										
		a	b	c								
1)	let f be the function defined by $f(x) = \ln(1 - \ln x)$ the domain of definition of f is :	$]0 ; +\infty[$	$]0 ; e]$	$]0 ; e[$								
2)	let $F(x) = \int_0^{2x} \frac{1}{\sqrt{t^2 + 1}} dt$ defined over \mathbb{R} then F(x)	Is increasing	Is decreasing	Is not monotone								
3)	Let g be a function defined for all real number x and $g(4-x) + g(x) = 10$. Then the curve of g has center of symmetry the point...	(4 ; 5)	(2 ; 10)	(2 ; 5)								
4)	Given the following table of variation of a continuous function h that is defined over \mathbb{R} . <table style="margin-left: 20px; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">x</td> <td style="padding: 5px;">$-\infty$</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">$+\infty$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">$h(x)$</td> <td style="padding: 5px;">-3</td> <td style="padding: 5px;">-5</td> <td style="padding: 5px;">$+\infty$</td> </tr> </table> <p style="margin-left: 20px;">The equation $h(x) = 1$ has</p>	x	$-\infty$	-1	$+\infty$	$h(x)$	-3	-5	$+\infty$	One solution	Two solutions	no solutions
x	$-\infty$	-1	$+\infty$									
$h(x)$	-3	-5	$+\infty$									

Exercise 2 (5 points)

The complex plane is attached to a direct orthonormal system $(O; \vec{u}; \vec{v})$

Given the points A, B and M of affixes: $z_A = 2i$, $z_B = \sqrt{3} - i$, $z_M = 2ie^{i\theta}$, $\theta \in]0; 2\pi[$

1) In this part suppose that $\theta = \frac{2\pi}{3}$

a) Find the exponential form of z_B and z_M

b) Verify that $\frac{z_M - z_A}{z_B - z_A} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$. Deduce the exact nature of triangle ABM

2) let M' be a point of the plane such that $z_{M'} = \frac{2iz_M - 4}{z_M}$

a) Interpret geometrically $|z_{M'} - 2i|$ and verify that $|z| = 2$.

b) Show that $AM' \times OM = 4$ then deduce that if $\theta \in]0; 2\pi[$, the point M' varies over a circle (C), whose center and radius are to be determined.

Exercise IV (8 points)

Consider the two functions f and g that are defined on $]0 ; +\infty [$ by: $f(x) = (x - 1)(\ln(x) - 1)$ and $g(x) = \ln(x) - \frac{1}{x}$. Designate by (C) the representative curve of f in an orthonormal system.(unit:2cm)

1) a) Find $\lim_{x \rightarrow 0^+} g(x)$ and $\lim_{x \rightarrow +\infty} g(x)$.

b) Calculate $g'(x)$, then Setup the table of variation of g over $]0 ; +\infty[$.

c) Show that the equation $g(x)=0$ has a unique solution α . Verify that $1.75 < \alpha < 1.77$.

2) a) Calculate $\lim_{x \rightarrow 0^+} f(x)$ and $\lim_{x \rightarrow +\infty} f(x)$. Deduce an asymptote to (C).

b) Show that $f'(x) = g(x)$, then deduce that f is decreasing over $]0 ; \alpha[$ and increasing over $]\alpha ; +\infty[$.

c) Setup the table of variation of f . (take $\alpha = 1.76$)

d) Calculate $f(1)$ and $f(e)$. Draw (C)

3) a) Show that the function F defined on $]0 ; +\infty [$ by $F(x) = \left(\frac{1}{2}x^2 - x\right)\ln x - \frac{3}{4}x^2 + 2x$, is an antiderivative of f over $]0 ; +\infty [$.

b) Deduce the exact value of the area of the region of the plane bounded by (C), $x'Ox$, and the vertical lines of equations $x=1$ and $x=e$.

Exercise 3 (5 points)

Consider 2 urns U and V such that :

U contains 3 red balls and 2 white balls ; V contains 2 red balls and 3 white balls , we randomly draw one ball from U and put it in V , then we randomly draw 2 balls from V . Consider the following events:

R : "the drawn ball from U is red" .

B : "the drawn ball from U is white" .

A : "the 2 balls drawn from V are red" .

C : "the 2 balls drawn from V are white" .

1) Show that $P(A \cap R) = 0.3$ and calculate $P(A \cap B)$, deduce $P(A)$.

2) Calculate $P(C)$.

3) The drawn ball from U is white, calculate the probability of obtaining 1 red ball and 1 white ball from V .

4) Let X be the random variable equals to the number of red balls drawn from V .

a) Show that $P(X = 0) = 0.28$.

b) Determine the probability distribution of X .

c) Calculate $E(X)$.