econd exercise Compound pendulum

A compound pendulum is formed of a rod AB of negligible mass, which can rotate without friction in a vertical plane around a horizontal axis (Δ) passing through a point O of the rod so that OB = d. A particle of mass M is fixed at point B and another particle C of mass m < M, which can slide on the part OA of the rod is placed at a distance OC = x of adjustable value. Let a = OG be the distance between O and the center of gravity G of the pendulum (Fig.1). The gravitational potential energy reference is the horizontal plane containing O.

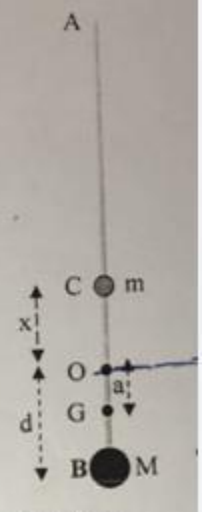


Fig.1

$g = 10 m/s^2$; $\pi^2 = 10$; $\sin\theta = \theta$ and $\cos\theta = 1 - \frac{\theta^2}{2}$, ($\theta$ in rad) for $\theta < 10^0$.

A- Theoretical study

1- Show that the position of G is given by: $a = \frac{Md - mx}{(M+m)}$.

2- Find the expression of the moment of inertia I of the pendulum about the axis (Δ) in terms of m, x, M and d.

3- The pendulum thus formed is deviated by an angle θ_0 from its equilibrium position and then released from rest at the instant $t_0 = 0$. The pendulum then oscillates around the stable equilibrium position. At an instant t, the position of the pendulum is defined by the angular abscissa θ, the angle that the vertical through O makes with OG, and its angular velocity is $\theta' = \frac{d\theta}{dt}$.

a) Write, at the instant t, the expression of the kinetic energy of the pendulum in terms of I and θ'.

b) Show that the expression of the gravitational potential energy of the system (pendulum, Earth) is $P.E = -(M+m) g a \cos\theta$.

c) Write the expression of the mechanical energy of the system (pendulum, Earth) in terms of M, m, g, a, θ, I and θ'.

d) Derive the second order differential equation in θ that governs the motion of the pendulum.

e) Deduce that the expression of the proper period, for oscillations of small amplitude, has the form: $T = 2\pi\sqrt{\frac{I}{(M+m)ga}}$.

f) Find the expression of the period T, in terms of M, m, d, g and x.

B- Application: metronome

A metronome is an instrument that allows adjusting the speed at which music is played.

The compound pendulum studied in part **A** represents a metronome where M = 50 g, m = 5 g, and d = 2 cm. The graph of figure 2 represents the variations of the period T of this metronome as a function of the distance x.

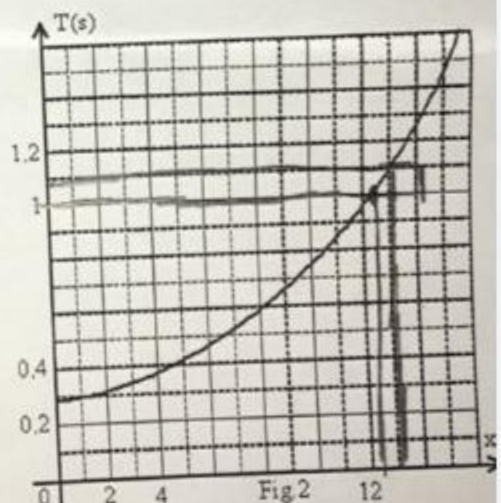


Fig 2

1) Find, in this case, the expression of the period T of the metronome as a function of x.

2) The leader of the orchestra (conductor), using a metronome to play a distribution, changes the position of C along OA, to follow the rhythm of the musical piece.

The rhythm is indicated by terms inherited from Italian for the classical distribution:

Name	Indication	Period (in s[illegible]
Grave	very slow	T = 1.5
Lento	Slow	1 ≤ T ≤ 1.[illegible]
Moderato	Moderate	0.6 ≤ T ≤ 0.[illegible]
Prestissimo	very fast	0.28 ≤ T ≤ 0.[illegible]

The Determine, using a method of your choice, the positions between which the leader of the orchestra may move C to adjust the speed to the rhythm **Lento**.

a) Determine the mechanical energy lost by the system (puck, two springs, Earth) between the instants $t_0 = 0$ and $t = 4.04$ s.

b) Deduce the average power lost in this interval.

3) The extremity A of the left spring is coupled to an exciter (E) of adjustable frequency « f » (Fig.4).
With an appreciable amount of friction, the puck is forced to oscillate on the air table with a frequency equal to that of (E). The variations, as a function of time, of the abscissa x of G is represented for two values of « f » by figures 5 and 6.

a) Determine, in each case, the amplitude and the period of the oscillations of G.

b) The amplitude of the oscillations represented in figure 6 is larger than that of the oscillations of figure 5. Interpret this increase.

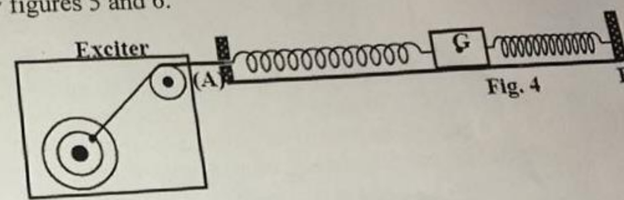
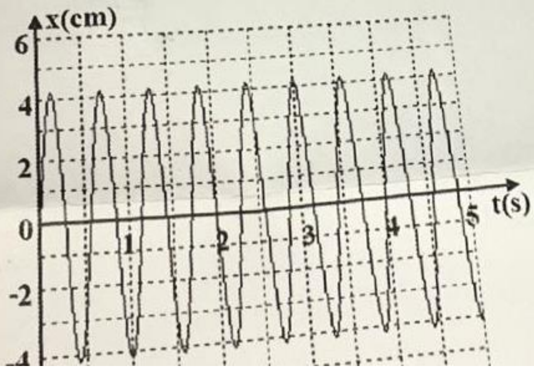


Fig. 4



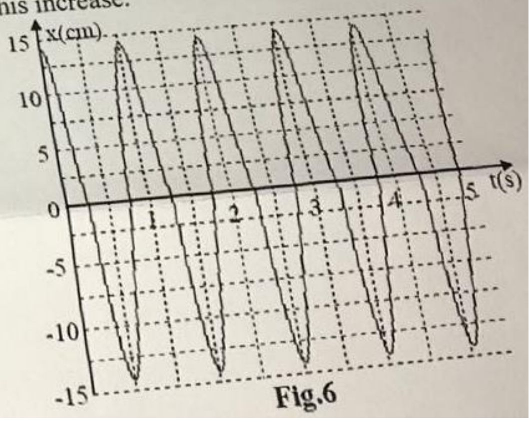


Fig.6