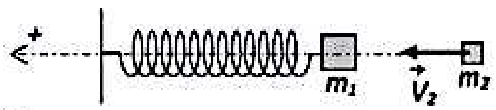
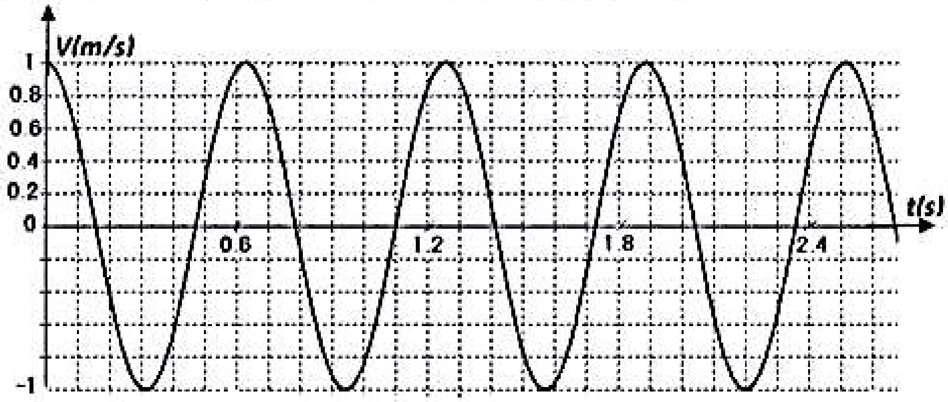
**First Exercise : Horizontal mechanical oscillator [ 11 pts.]**

An elastic spring of stiffness $k = 50N/m$ and of negligible mass is placed horizontally on an air table has one end fixed to a support while a block of mass $m_1 = 400g$ is connected to the free end.




A particle of mass $m_2 = 100g$ is shot on $m_1$ with a velocity $\vec{V}_2$ and is stuck to it thus forming one solid that will be considered as a particle of mass $M = 500g$. We neglect friction and take $g = 10m/s^2$ and take $\pi^2 = 10$.

The adjacent diagram shows the variation of the velocity of the center of mass $G$ of $M$ during its oscillations. The instant $t = 0$ is the instant of impact between $m_1$ and $m_2$.




A. **Collision between $m_1$ and $m_2$**

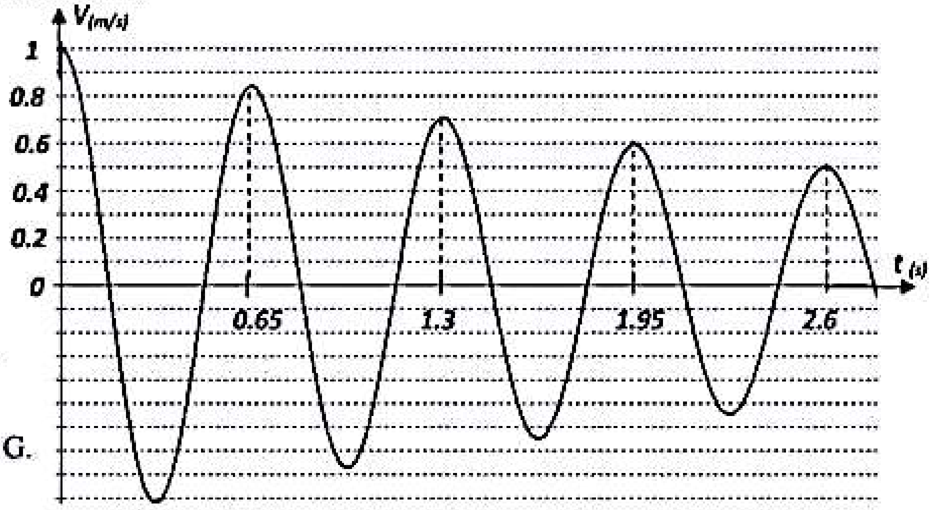
1. Use the adjacent graph to give the speed of $M$ right after impact.
2. Determine the speed $V_2$ of $m_2$ just before impact.
3. Is this collision elastic? Justify.
4. Determine the maximum compression $Xm$ of the spring.

**B. Horizontal harmonic oscillator.**

1. Give the expression of the mechanical energy of the system [*oscillator, Earth*], taking the horizontal level of the center of mass of $M$ as a gravitational potential energy reference. Calculate its value at the instant t = 0.
2. Derive the differential equation that governs the motion of $G$. deduce the expression of its angular frequency.
3. Calculate the proper period of oscillations and compare it with the period of the above graph.
4. Write down the time equation of motion of $G$ and deduce the expression of its velocity.
5. How does the expression of the velocity agree with the above graph?
6. What is the mode of oscillations thus obtained?

**C. Different mode of oscillation**

The functioning of the air table is no longer ideal and a force of friction of the form $\vec{f} = -\lambda\vec{V}$, as a result, the variation of the velocity of $G$ is as shown in the adjacent figure.




1. What is the mode of oscillations thus formed?
2. Derive, using non- conservation of mechanical energy, the differential equation that governs the motion of G.
3. Determine, using the graph the pseudo period , compare it with the

proper period.

*4.* Calculate the variation of mechanical energy between $t = 0$ and $t = 2.6s$.

*5.* It is required to *drive* the oscillations of this oscillator.

*i)* What is meant by *driving* these oscillations?

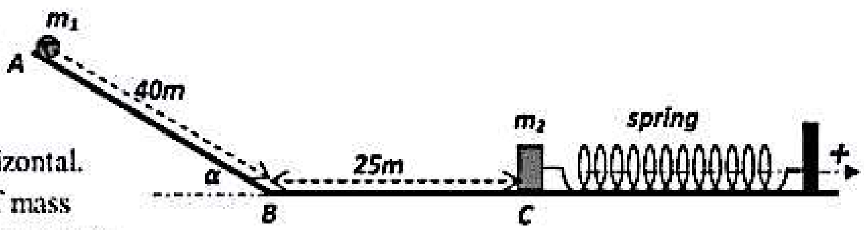
*ii)* Calculate the average power of the *driving* system.

***Second Exercise*** ***Mechanical energy and collision*** *[ 7 pts.]*

A particle of mass $m_1 = 100g$ is released at the point $A$ of the track $ABC$ along which it may slide without rolling.

The part $AB = 40m$ is frictionless and is inclined by an angle $\alpha = 30^0$ with the horizontal.

At the end point $C$ of this track, a block of mass $m_2 = 200g$ is connected to the free end of an elastic spring of stiffness $k = 80N/m$.




The horizontal plane containing $BC = 25m$ is taken as a gravitational potential energy reference.

***I.***

***Motion along AB***

*a)* Calculate the mechanical energy of the system [$m_1$, *Earth*] at $A$.

*b)* The mechanical energy of this system is conserved. Justify.

*c)* Calculate the speed of $m_1$ at $B$.

***II.***

***Motion along BC.*** The particle $m_1$ slides along $BC$ and reaches $C$ with the speed $V_1 = 15m/s$

*a)* Calculate the mechanical energy of the system [$m_1$, *Earth*] upon reaching point $C$.

*b)* The mechanical energy of this system is not conserved over $BC$. Justify.

*c)* Calculate the force of friction along $BC$, assumed constant.

***III.***

***Collision between $m_1$ and $m_2$***. The particle $m_1$ enters in a head-on elastic collision with $m_2$.

*a)* Determine the velocities $V_1'$ and $V_2'$ of $m_1$ and $m_2$ respectively right after impact.

*b)* If the impact lasted $0.05s$ what is the average force received by each during impact?

*c)* Does $m_1$ rebounds after impact? Justify and determine the point where it stops for the first time after impact.

***IV.***

***Mechanical oscillator.*** After impact, the system [$m_2$, *spring*] form a horizontal elastic pendulum whose motion is sinusoidal and whose proper period is given by $T_o = 2\pi\sqrt{\frac{m}{k}}$

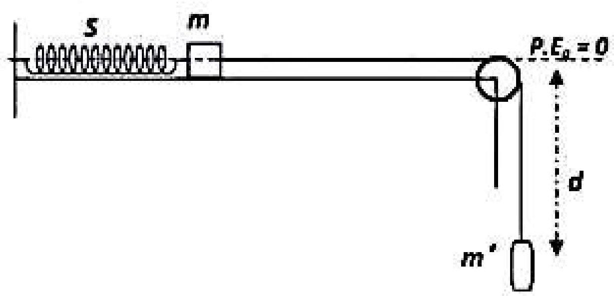
*a)* Calculate the amplitude of the oscillations.

*b)* Calculate the time taken by $m_2$ to stop for the first time after impact.

***Third Exercise*** *[15 pts]*

## A. *Harmonic oscillator*

Consider the oscillating system shown in the adjacent figure. An elastic spring of stiffness $k = 100N/m$ has one end fixed to a support while the other end is connected



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to a mass $m = 600g$. A string of negligible mass passing over a light pulley connects $m$ to another mass $m' = 400g$ that hangs vertically. Take $g = 10m/s^2$. Neglect friction.

The horizontal plane through the enter of mass $G$ of $m$ is taken as a gravitational potential energy reference. At equilibrium, the spring is elongated by a distance $\Delta\ell$ and the center of mass of $m'$ is at $d = 50cm$ below the level of $P.E_g$ reference. The position of the center of mass of $m$ is now at a point $O$ taken as an origin of abscissa.

*1)* Show that $\Delta\ell = 4cm$

*2)* $m'$ is pushed down by $2cm$ then released without initial velocity at $t = 0$

*a)* Find, when $m'$ is in the lowest position, the mechanical energy of the system [$S$, $m$, $m'$, *Earth*].

*b)* Find, at any time $t$, the mechanical energy of the system in terms of masses, the abscissa of $G$ and the speed $V$ of the masses and the stiffness $K$ of the spring.

*c)* Find the maximum speed of $m$

*d)* Derive the differential equation that governs the motion of $G$ and calculate its period.

*e)* When $m'$ is in the lowest position, the string is cut. Describe in details the subsequent motion of $G$ and determine its new period and new amplitude and the new maximum speed of $m$

## B. Elastic Pendulum

An air puck, of mass m = 100 g, is attached to a spring of constant k and of negligible mass; the other extremity of the spring is fixed. The air puck can slide on a horizontal straight rail. Thus, we obtain a frictionless horizontal elastic pendulum.

A certain electric set up allows a computer to record the motion of the center of inertia G of the air puck in terms of time. The position of G is located with its abscissa x on an axis (O, $\vec{i}$) where O coincides with the equilibrium position of G. The air puck is put in motion; we perform then two experiments with the same spring and the same air puck.

The two figures 1 and 2 show the recordings of these two experiments.

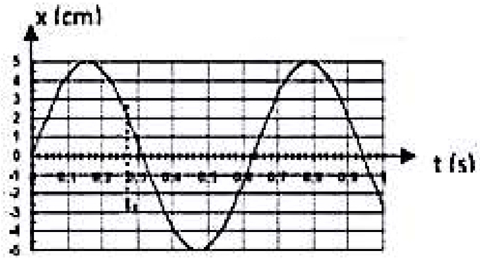

Fig.1

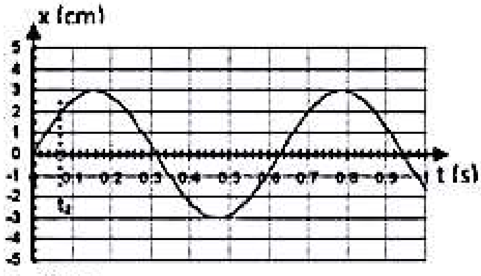

Fig.2

Attention: all answers must be justified.

A. Kinetic study

1. Which one of the three following expressions of the natural period To is not compatible with the unit of To?

a. $To = 2\pi\sqrt{\frac{k}{m}}$ b. $To = 2\pi\sqrt{\frac{m}{k}}$ c. $To = 2\pi\sqrt{\frac{x_o}{g}}$

Where xo is the amplitude of oscillations and g the gravitational constant.

2. Using the two curves (Fig. 1 and Fig. 2), calculate the value of To in each experiment and specify which one of the two remaining expressions is correct.

3. Calculate the value of the angular frequency ωo of oscillations and deduce the constant k of the spring.

4. We propose three time equations to describe the motion of the air puck in the first recording (Fig. 1):

$x = 5\sin(10\,t)$; $x = 5\sin(0.63\,t)$; $x = 3\sin(0.63\,t)$ [x : cm ; t : s]

a. Which equation is compatible with the recording?

b. Write down the time equation of the velocity V of the air puck in terms of time t. Deduce Vo, the maximum value of V.

B. Energetic analysis

1. From the results of experiment (1), the computer draws the curve describing the variation of one form of energy in terms of time (Fig.3).

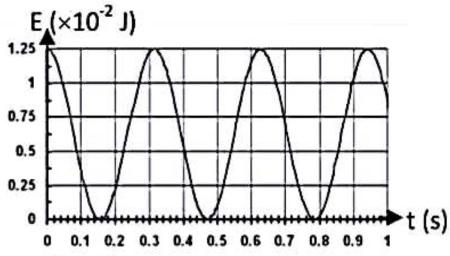

Fig. 3

a. Does the curve E = E (t) represent the variations of the mechanical energy of the system S (spring, air puck), or its elastic potential energy or its kinetic energy?

b. The mechanical energy of the system S is supposed to be constant. Using the graph of figure 3, mention an experimental observation which justifies this hypothesis.

c. 1°- Express the mechanical energy in terms of m, V, k and x.

2°- Does the figure 4 represent the variation of V2 in terms of x2?

3°- What are the numerical values of a and b of figure 4?

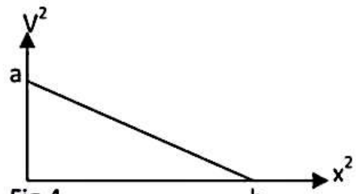

Fig.4

C. Comparison of the results of the two experiments

We choose two times t1 and t2 for which x is equal to 2 cm in the two experiments. (Figures 1 and 2). Reproduce, the following table and complete it by putting in the comparison column one of the following signs (<, >, =) and making justification in the corresponding column.

| | First experiment | comparison | second experiment | justifications |
|---|---|---|---|---|
| Potential energy of S | EPE1 | = | EPE2 | The EPE depends on the elongation x which is the same in the two experiments, thus EPE1 = EPE2 |
| Mechanical energy of S | ME1 | | ME2 | |
| Kinetic energy of S | KE1 | | KE2 | |
| Velocity of the air puck | V1 | | V2 | |

***Fourth exercise: [3pts]***

A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 2000 N/m, as shown in the figure below. The spring is unstretched when the system is as shown in figure (a) [the system is not in equilibrium yet], and the incline is frictionless.

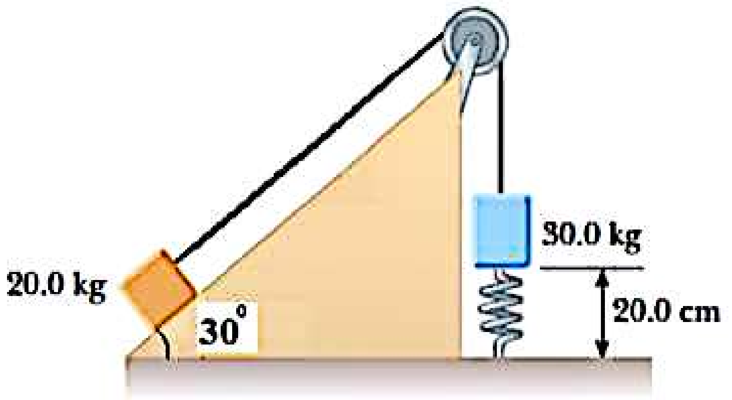


fig (a)

a) The 20-kg block ascends and the 30-kg block descends so that they reach equilibrium when they are on the same horizontal level as figure (b) shows. Find the compression in the spring at equilibrium.
b) The 30-kg block is displaced 5 cm downward then left free from rest as shown in figure (c), find the mechanical energy of the system (the two blocks, spring, Earth) at any instant of time taking the horizontal level passing through the equilibrium position as the gravitational potential energy level.

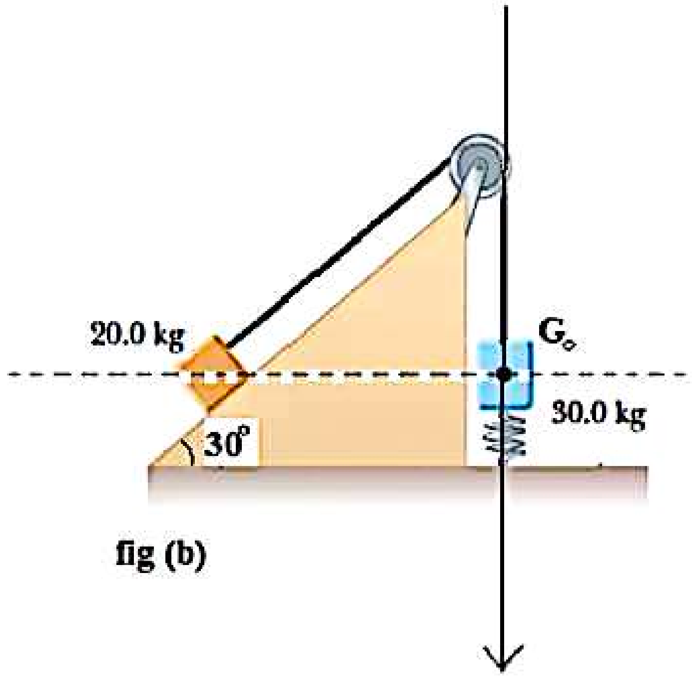


fig (b)

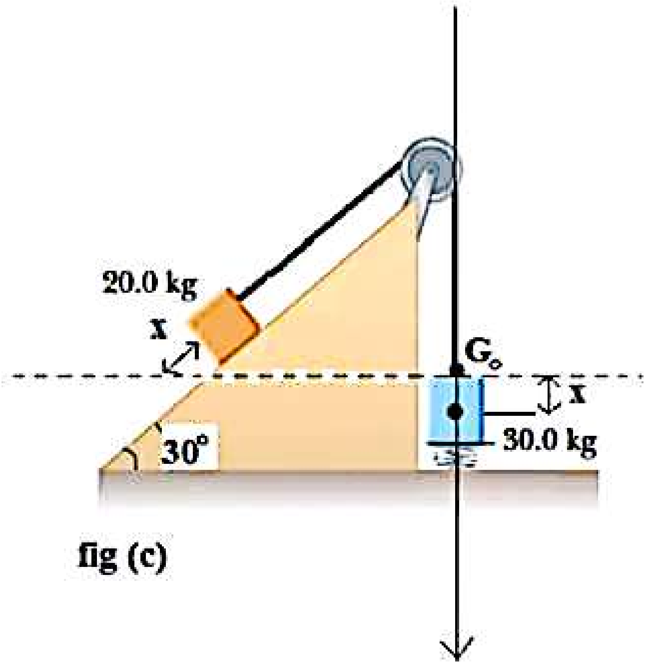


fig (c)

c) Derive the second order differential equation of motion.
d) The solution of the differential equation is given by: $x = X_m \cos(\omega_o + \varphi)$. Find $X_m$, $\omega_o$ and $\varphi$.