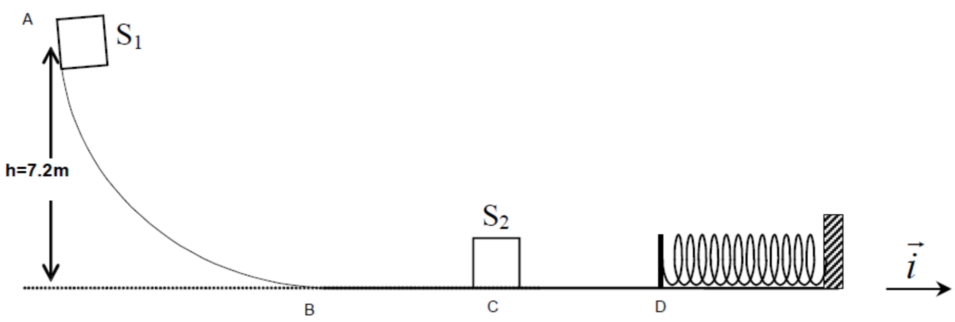
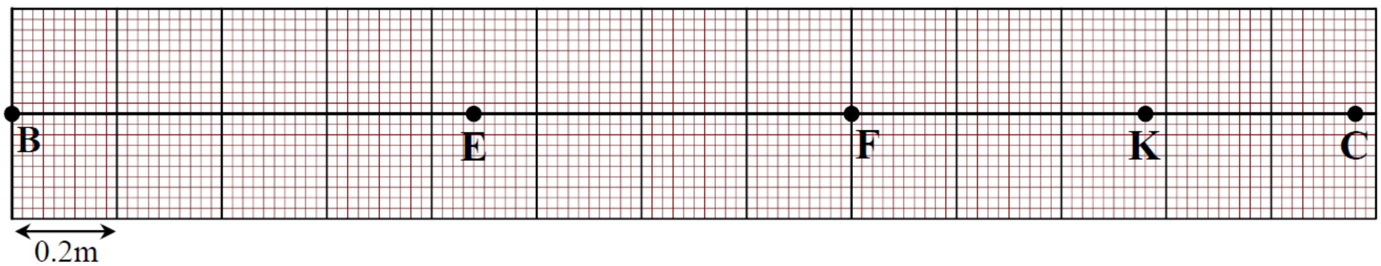
Exercise one: **(7points)**

Mechanics

A particle S_1 of mass $m_1 = 0.2kg$ is released from rest at a point A to slide without friction along a path AB. The portion BC of the path exerts a force of friction of a constant magnitude f_r while the path beyond point C is again frictionless. At point C, another particle S_2 of mass $m_2 = 0.3kg$ is placed at rest. After moving along BC, S_1 enters in a head-on collision with S_2 and sticks to it thus forming one particle S. S then continues until it strikes the free extremity of a spring R of unjointed turns and spring constant K. As a result, R is compressed by $x_m = 20cm$. The horizontal along B is taken as a reference of gravitational potential energy. Use $g = 10m/s^2$.





A) Moving along AB:

1. Write the expression of the mechanical energy of S_1 at any instant along AB. Calculate its value.
2. What can be said about this mechanical energy? Justify.
3. Determine the height of S_1 when both of its kinetic and potential energy are equal. Calculate its speed at that moment.
4. Determine its velocity at the bottom B of the path.

B) Moving along BC:

By some techniques, the motion of S_1 along BC is registered and its consequence positions are given by the dot prints shown above. The time interval between two successive dot prints is $\tau = 80$ ms.

1. Determine the velocities $\vec{V}_E$ and $\vec{V}_K$ of S_1 at E and K.
2. Determine the linear momentum vectors $\vec{P}_E$ and $\vec{P}_K$ of S_1 at these two points.
3. Calculate the variation $\Delta\vec{P}$ of linear momentum of S_1 as it moves from E to K.
4. Apply Newton's second law $\sum \vec{F}_{ext} = \frac{\Delta \vec{P}}{\Delta t}$ to determine the magnitude f_r of the force of friction.
5. Determine the velocity $\vec{v}$ at which S_1 hits S_2.

C) Moving along CD:

S_1 and S_2 being stuck after collision, the formed system S moves horizontally along a frictionless path at a velocity of magnitude V_G.

1. Calculate V_G. Specify the applied law in this case.
2. Calculate the mechanical energy of S at the moment it hits R. Deduce its potential elastic energy

when the R is in maximum compression state.

3. Calculate K.

<u>Exercise two:</u> (7 points)

Studying the motion of an elastic horizontal pendulum

A horizontal elastic pendulum is formed of solid (S), of mass 1.08kg, that is connected at one end of mass less spring of unjointed turns and of stiffness K [figure (1)]. The system thus formed (S, spring) is made to oscillate on a frictionless table. While oscillating, (S) is connected to a convenient computer interface that traces the variation of the elastic potential energy as a function of time [figure (2)] is obtained. The horizontal level through the center of mass is taken as a reference for gravitational potential energy. The initial velocity vector is $\vec{V_0} = V_0 \vec{i}$. Take g = 10 m/s^2 and π^2 = 10.

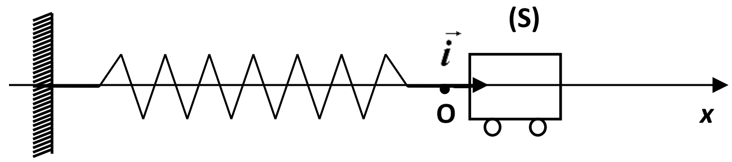


Figure (1)

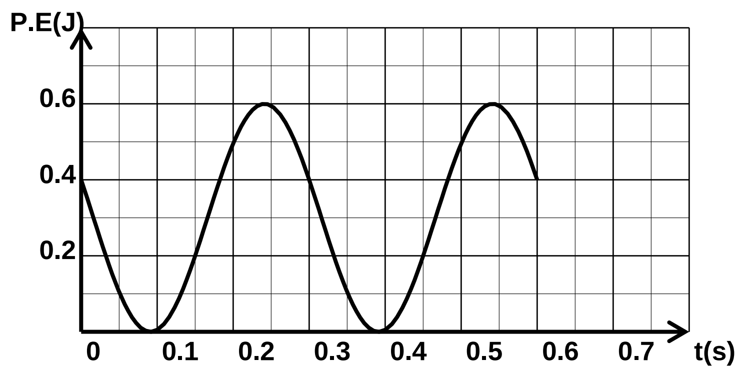


Figure (2)

A- <u>Study of energy.</u>

1. Write the expression of the mechanical energy of the oscillator at any instant in terms of ***V, K and x***.

2. Use the graph of figure (2), calculate, with justification, the value of the mechanical energy of the system.

3. Calculate the value of the kinetic energy at t = 0. Deduce the value of V_0.

4. Determine when (S) passes through the equilibrium position at point O for the first time:

- **a.** The elastic potential energy, the mechanical energy and the kinetic energy of the system.
- **b.** The instant t.
- **c.** The velocity vector $\vec{V}$.

B- Study of the motion.

1. Derive the expression of the differential equation that describes the motion of the system. Specify the nature of motion.

2. Determine, referring to figure (2), the proper period T_0 of the motion. Deduce the value of ω_0 ***and K***.

3. The time equation is given $x = x_m \cos(\omega_0 t + \varphi)$. Determine x_m.

4. Determine at t = 0, the abscissa x_0 of (S). Deduce the value of φ.

5. Verify that the speed of (S) at any instant is: $V_{(m/s)} = \cos(\omega_0 t + \frac{7\pi}{10})$.

C- Variation of linear momentum of (S):

At the instant t=1 sec, (S) is released from the spring and continues horizontally at a constant velocity $\vec{v}_1 = \vec{i}$ m/s. At that velocity, (S) undergoes a sudden impact with a wall, that lasts for 50ms and rebounds back at v_2= 0.6 m/s.

1. Calculate the linear momentum of (S) before and after impact. Deduce the variation in linear momentum $\Delta\vec{P}$.

2. Apply Newton's second law in terms of linear momentum; determine the force $\vec{F}$ exerted by the wall on (S).

3. Determine the energy absorbed due to impact.

Correction of Exam			
Exercise one			**Points**
A)1	M.E = K.E + P.E = ½ m_1v^2 + m_1gh At A, v_A=0 and h=7.2m ⇒ M.E = m_1gh= 0.2x10x7.2 =14.4J		0.25 0.25
A)2	Since no friction ⇒ Mechanical energy is conserved.		0.25
A)3	$K.E = P.E = \frac{M.E}{2} = 7.2J$ $P.E = m_1gh \Rightarrow h = 3.6m$ and $K.E = \frac{1}{2}m_1v^2 \Rightarrow v = \sqrt{72}m/s$		0.25 0.5
A)4	$M.E_A = M.E_B \Rightarrow 14.4 = K.E_B + P.E_B \Rightarrow \frac{1}{2}m_1v_B^2 + 0 = 14.4 \Rightarrow v_B = 12m/s$		0.5
B)1	$v_E = \frac{BF}{2\tau} = \frac{8div \times 0.2m/div}{2\times 0.08} = 10m/s \Rightarrow \vec{v}_E = 10\vec{i}$ $v_K = \frac{FC}{2\tau} = \frac{4.8div \times 0.2m/div}{2\times 0.08} = 6m/s \Rightarrow \vec{v}_K = 6\vec{i}$		0.25 0.25
B)2	$\vec{P}_E = m_1\vec{v}_E \Rightarrow \vec{P}_E = 0.2\times 10\vec{i} = 2\vec{i}$ and $\vec{P}_K = m_1\vec{v}_K \Rightarrow \vec{P}_K = 0.2\times 6\vec{i} = 1.2\vec{i}$; P in kgm/s.		0.75
B)3	$\Delta\vec{P} = \vec{P}_K - \vec{P}_E = 1.2\vec{i} - 2\vec{i} = -0.8\vec{i}$ (Kgm/s)		0.25
B)4	Forces acting on S_1 along BC are: its weight $m\vec{g}$, normal reaction $\vec{N}$ and the force of friction $\vec{f}_r$. Apply Newton's second law: $\sum\vec{F}_{ext} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m_1\vec{g} + \vec{N} + \vec{f_r} \Rightarrow -f_r\vec{i} = \frac{-0.8\vec{i}}{0.16} \Rightarrow f_r = 5N$		0.25 0.5
B)5	$\sum\overrightarrow{F_{ext}} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow -5\vec{i} = \frac{\vec{P}_C - \vec{P}_K}{0.16} \Rightarrow -0.8\vec{i} = 0.2\vec{v}_C - 1.2\vec{i} \Rightarrow \vec{v}_C = 4\vec{i} \Rightarrow v_C = 4m/s$		0.5
C)1	During collision: $\sum\overrightarrow{F_{ext}} = \vec{0}$ then linear momentum of the system is conserved. $\vec{P}_b = \vec{P}_a \Rightarrow m_1\vec{v} = (m_1 + m_2)\vec{V}_G \Rightarrow \vec{V}_G = 1.6\vec{i}$ m/s		 0.5

C)2	When x=o; V= 1.6m/s $\Rightarrow M.E=\frac{1}{2}(m_1+m_2)V_G^2=0.64J$ $M.E \text{ is conserved} \Rightarrow M.E_{(x=0)}=M.E_{(x=x_m)}=0.64J$	0.25 0.25
C)3	$M.E_{(x=x_m)}=\frac{1}{2}Kx_m^2 \Rightarrow K=32N/m$	0.5
Exercise two		**Points**
A)1	$M.E(t)=K.E+P.E_e+P.E_g=\frac{1}{2}mv^2+\frac{1}{2}kx^2$	
A)2	No friction, then mechanical energy of the system is conserved. Graphically: M.E= $P.E_{e(max)}$=0.6J	
A)3	At t =0: $P.E_e$=0.4J But K.E=M.E –$P.E_e$ $\Rightarrow$$K.E_{(at\ t=0)}$=0.6-0.4=0.2J Also $K.E_{(at\,t=0)}=\frac{1}{2}mv_0^2 \Rightarrow v_0=0.61m/s.$	
A)4-a	$x=0$ for thefirst time$:P.E_e=0J;\ M.E=0.6J$ and $K.E=M.E-P.E_e=0.6J$	
A)4-b	When x=0 for the first time: t=0.09s (from the graph)	
A)4-c	When x=0 for the first time: $K.E=\frac{1}{2}mv^2 \Rightarrow v=\mp1.054m/s \Rightarrow \vec{v}=-1.054\vec{i}$	
B)1	$M.E=\frac{1}{2}mv^2+\frac{1}{2}kx^2$ $\frac{dM.E}{dt}=0 \Rightarrow mvv'+kxx'=0$ but $v'=x''$ and $x'=v$ $\Rightarrow x''+\frac{k}{m}x=0$ The differential equation has the for$:x''+\omega_0^2x=0 \Rightarrow$ The motion of (S) is simple harmonic such that$:\omega_0=\sqrt{\frac{k}{m}}$	
B)2	From graph: period of energy T=0.3 s but T_0 = 2 T = 0.6s $\omega_0=\frac{2\pi}{T_0}=\frac{10\pi}{3}rd/s.$ But $\omega_0=\sqrt{\frac{k}{m}} \Rightarrow k=\omega_0^2m=120N/m$	

B)3	When x = x_m; v = 0 $\Rightarrow M.E = \frac{1}{2}kx_m^2 \Rightarrow x_m = 0.1m$	
B)4	$at\ t=0: P.E_e = \frac{1}{2}kx_0^2 \Rightarrow 0.4 = 0.5\times120x_0^2 \Rightarrow x_0 = \mp0.0282m$ But $P.E_e$ is decreasing and v<0 then x>0 so that x =0.082m $At\ t=0: x = 0.082m\ and\ v<0 \Rightarrow 0.082 = 0.1\cos\varphi \Rightarrow \mp0.62rd = \mp\frac{\pi}{5}rd$ $But: v = -x_m\omega_0\sin(\omega_0 t+\varphi);\ at\ t=0: -x_m\omega_0\sin\varphi<0 \Rightarrow \sin\varphi>0 \Rightarrow \varphi = \frac{\pi}{5}rd$	
B)5	$v = -x_m\omega_0\sin(\omega_0 t+\varphi) = -0.1\times\frac{10\pi}{3}\sin(\omega_0 t+\frac{\pi}{5}) = -\sin(\omega_0 t+\frac{\pi}{5}) = \sin(\omega_0 t+\frac{\pi}{5}+\frac{\pi}{2})$ $\Rightarrow v = \sin(\omega_0 t+\frac{7\pi}{10})$	
C)1	$\vec{P}_b = m\vec{v}_1 = 1.08\vec{i}$ (kgm/s). Since (S) rebounds back $\Rightarrow$ $\vec{v}_2 = -0.6\vec{i} \Rightarrow \vec{P}_a = -0.648\vec{i}$ $\Delta\vec{P} = \vec{P}_a - \vec{P}_b = -1.728\vec{i}$	
C)2	$\sum\vec{F}_{ext} = \frac{\Delta\vec{P}}{\Delta t} \Rightarrow m\vec{g}+\vec{N}+\vec{F} = \frac{-1.728\vec{i}}{0.05} \Rightarrow \vec{F} = -34.56\vec{i}$	
C)3	$E_{absorbed} = K.E_b - K.E_a = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2 = 0.31J$	
		Points
	At any instant Mechanical energy of the system is: M.E= K.E +P,E_e + P.E_g $M.E = \frac{1}{2}Mv^2 + \frac{1}{2}kx^2$ $when: x = x_m; v=0 \Rightarrow M.E = \frac{1}{2}kx_m^2$ $when\ x=o; v=v_C \Rightarrow M.E = \frac{1}{2}Mv_C^2$ $But\ mechanical energy\ is\ conserved,\ then:$ $\frac{1}{2}kx_m^2 = \frac{1}{2}Mv_C^2 \Rightarrow v_C = 2.84m/s$	

	$\frac{dM.E}{dt}=0\Rightarrow Mvv'+kxx'=0\ but\ v'=x''\ and\ x'=v\neq 0$ $Then: x''+\frac{k}{M}x=0$	
	At t=o x=0 ⇒0 = $x_m$sinφ ⇒ ω= 0 or πrd $v=x'=\omega_0 x_m\cos(\omega_0 t+\varphi)$ $at\ t=0: v=v_C>0\Rightarrow \omega_0 x_m\cos\varphi>0\Rightarrow\cos\varphi>0\Rightarrow\varphi=0$ $The\ form\ of\ the\ differential\ equation\ is:$ $x''+\omega_0^2 x=0\Rightarrow\omega_0^2=\frac{K}{M}\Rightarrow\omega_0=2rd/s$ $but\ T_0=\frac{2\pi}{\omega_0}=3.14s$	
	Figure (1)	
	$\vec{P}_b=\vec{0}\ and\ \vec{P}_a=M\vec{v}_C+m\vec{v}_G\neq 0\Rightarrow\vec{P}_b\neq\vec{P}_a$ Then linear momentum of the system (C, shell) is not conserved.	
	The system [(C) shell] is subject to the normal reaction of the rails (no friction) which is vertical and its weight is too vertical, so: $(M+m)\vec{g}+\vec{N}=\frac{d\vec{P}}{dt}\Rightarrow\frac{dP_x}{dt}=0\Rightarrow\Delta P_x=0\Rightarrow P_{ax}-P_{bx}=0\Rightarrow$ $mv_{0x}+Mv_C=0\Rightarrow v_C=-\frac{m}{M}v_{0x}=-\frac{200}{5000}(-100\cos 45^0)=2.83m/s$	
	Figure (b): $\lambda_2=1.5\times10^4$kg/s	
	For figure (a) $\lambda_1=5\times10^4$kg/s; little damping For figure (c) $\lambda_3=3\times10^4$kg/s; large damping	
	T=3.25 s	
	T >T_0 Due to damping effect.	